1128
B.Sc. (Hons.) Biotechnology
First Semester
BIOT-Sem-I-III-T (a): Mathematics

Time allowed: 3 Hours

Max. Marks: 67

NOTE: Attempt _five_ questions in all, including Question No. I (Section-A) which is compulsory and selecting two questions each from Section B-C.

x-x-x

Section-A

1. (a) _Prove that_ $\sqrt{3}$ _is an irrational number._

   (b) _Simplify_ : $\dfrac{(64)^{2/3} \times (81)^{3/4}}{(27)^{2/3}}$

   (c) _Express_ $\dfrac{5-\sqrt{3}}{2+\sqrt{3}}$ _in the form_ $a + b\sqrt{3}$, _where_ a, b _are integers._

   (5,4,4)

2. (a) _Convert_ $\dfrac{1+7i}{(2-i)^2}$ _in the Polar form._

   (b) If $x + iy = \dfrac{(a+i)^2}{2a^2+1}$, _prove that_ $x^2 + y^2 = \dfrac{(a^2+1)^2}{(2a^2+1)^2}$

   (c) If $\left(\dfrac{1}{1-2i} + \dfrac{3}{1+i}\right)\left(\dfrac{3+4i}{2-4i}\right) = a + ib$, _find a and b._

   (5,4,4)

3. (a) _A college awarded 38 medals in football, 15 in basketball and 20 in Cricket. If these medals went to a total of 58 students and only three students got medals in all the three sports, how many students received medals in exactly two of the three sports?_

   (b) _Let_ $A = \{1,2,3,4,5\}$. _Define a relation R on A defined by_ :
   $R = \{(x,y): y = 2x - 1\}$
   (i) _Write R in roster form._
   (ii) _Depict this relation using an arrow diagram._
   (iii) _Write down the domain, codomain and range of R._

   (c) _Draw the graph of the modulus function. Also, write its domain and range._

   (5,4,4)

P.T.O.

4. (a) *Evaluate:* $\lim_{x\to 0} \dfrac{(\sin 7x + 6x)}{(7x + \sin 6x)}$

(b) *Suppose* $f(x) = \begin{cases} 5 & , & x \leq 2 \\ ax + b & , & 2 < x < 10 \\ 21 & , & x \geq 10 \end{cases}$ *and if* $\lim_{x\to 2} f(x)$ *and* $\lim_{x\to 10} f(x)$

both exists, find the values of a and b.

(c) *Evaluate* : $\lim_{x\to 1} \left(\dfrac{2}{1-x^2} + \dfrac{1}{x-1} \right)$

(5,4,4)

Section-B

5. (a) *Find* $\dfrac{dy}{dx}$, *if* $y^x + x^y = a^b$

(b) *A balloon, which always remains spherical on inflation, is being inflated by pumping in* 900 cm^3 *of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15cm.*

(c) *Find local maximum and local minimum values of the function f given by* $f(x) = x^3 - 6x^2 + 9x + 15$

(5,4,4)

6. (a) *Evaluat* : $\int e^{2x} \sin x \, dx$

(b) *Find the area bounded by the curve* $x^2 = 4y$ *and the straight line* $x = 4y - 2$.

(c) *Solve* : $(x + y - 5) \dfrac{dx}{dy} = 1$

(5,4,4)

7. (a) Solve the following Linear Programming Problem graphically:

Maximize z = 1000x + 600y
Subject to : x + y ≤ 200
x ≥ 20
y ≥ 4x
x, y ≥ 0

(b) *In a study of diabetic patients, the following data was obtained:*

Age(in years)	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of patients	3	8	30	36	27	15	6

Draw the histogram and a frequency curve for the above data.

(7,6)

8. (a) *Calculate the mean, variance and standard deviation for the following frequecy distribution:*

Classes	0-30	30-60	60-90	90-120	120-150	150-180	180-210
Frequency	2	3	5	10	3	5	2

8. (b) *Calculate median, first quartile and 85th percentile for the following data:*

Classes	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
Frequency	75	250	350	192	68	35	24	6

(7,6)

Section – C
(Compulsory Questions)

9. (a) *Let $A = \{1,2,3,4,5\}$, $B = \{1,3,5\}$, Find $A - B$ and $B - A$.* (2)

(b) *Simplify :* $\dfrac{(5-4i)(4+5i)}{(4+3i)}$ (2)

(c) *Differentiate $e^{\sin^2 x}$ w.r.t $\sin x$.* (2)

(d) *Evaluate $\int \dfrac{\sqrt{\tan x}}{\sin x \cos x} dx$* (2)

(e) *Show that the function $f(x) = \log \sin x$ is strictly increasing on $\left(0, \frac{\pi}{2}\right)$.* (2)

(f) *Form a differential equation of the family of curves*
$\dfrac{x}{a} + \dfrac{y}{b} = 1$, *by eliminating the arbitary constants a and b.* (2)

(g) *Find the median, lower quartile and upper quartile of the following* (3)
numbers: $5, 22, 6, 3, 7, 33, 9$

x~x~x